647510 **BSCgraCC6010** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
April/May-2022 (OLD COURSE)
GRAPH THEORY AND COMPLEX ANALYSIS -2(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1

(A) ATTEMPT ANY ONE **07**

(1) Write a short note on $\boldsymbol{K\ddot{o}nigsberg\ Bridge}$ problem.

(2) Define: Connected Graph
Prove that a graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$ is disconnected $\boldsymbol{iff}$ its vertex set $\boldsymbol{V}$ can be partitioned into two nonempty, disjoint subsets $\boldsymbol{V_1}$& $\boldsymbol{V_2}$ such that there exists no edge in $\boldsymbol{G}$ whose one end vertex is in subset $\boldsymbol{V_1}$ and the other in subset $\boldsymbol{V_2}$.

(B) ATTEMPT ANY ONE **04**

(1) Define:(i) Subgraph (ii) Edge-disjoint Subgraphs (iii)Closed Walk (iv) Euler graph.

(2) A simple graph with $\boldsymbol{n}$ vertices and $\boldsymbol{k}$ components can have at most $\frac{(\boldsymbol{n}-\boldsymbol{k})(\boldsymbol{n}-\boldsymbol{k}+\boldsymbol{1})}{\boldsymbol{2}}$ edges.

(C) ATTEMPT ANY ONE **03**

(1) Define: (i) Isolated Vertex (ii) Pendant Vertex (iii) Null Graph

(2) Show that an infinite graph with a finite number of vertices will have at least one pair of vertices joined by an infinite number of parallel edges.

Que-2

(A) ATTEMPT ANY ONE **07**

(1) Define: Embedding , Planar Graph
Prove that Kuratowski's second graph is non-planar.

(2) State the Euler's Formula for planar graphs.
If $\boldsymbol{G}$ be a graph with n vertices, $\boldsymbol{e}$ edges and $\boldsymbol{f}$ faces, then prove that
(i) $\boldsymbol{e} \geq \frac{\boldsymbol{3}}{\boldsymbol{2}}\boldsymbol{f}$ (ii) $\boldsymbol{e} \leq \boldsymbol{3n} - \boldsymbol{6}$.

(B) ATTEMPT ANY ONE **04**

(1) Prove that the vertex connectivity of a graph can never exceed its edge connectivity.

(2) Prove that the rank of the incidence matrix of a connected graph $\boldsymbol{G}$ with $\boldsymbol{n}$ vertices is $\boldsymbol{n} - \boldsymbol{1}$.

(C) ATTEMPT ANY ONE **03**

(1) Define: Adjacency Matrix.
Write any four observations of the adjacency matrix of a graph.

(2) Define: (i)Proper Coloring (ii) Chromatic Number (iii) Independence Number

Que-3

(A) ATTEMPT ANY ONE **07**

(1) Explain: Conformal Mapping.

(2) Discuss the transformation $\boldsymbol{w} = \boldsymbol{cosz}$.

(B) ATTEMPT ANY ONE **04**

(1) Show that under the transformation $= \frac{\boldsymbol{1}}{\boldsymbol{z}}$, the images of the lines $\boldsymbol{y} = \boldsymbol{x} - \boldsymbol{1}$ and $\boldsymbol{y} = \boldsymbol{0}$ are the circle $\boldsymbol{u}^2 + \boldsymbol{v}^2 - \boldsymbol{u} - \boldsymbol{v} = \boldsymbol{0}$ and the line $\boldsymbol{v} = \boldsymbol{0}$ respectively.

(2) Show that Bilinear mapping $\boldsymbol{w} = \frac{\boldsymbol{az+b}}{\boldsymbol{cz+d}}$, $\boldsymbol{z} \neq \frac{-\boldsymbol{d}}{\boldsymbol{c}}$, $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ & $\boldsymbol{d} \in \mathbb{C}$, $\boldsymbol{ad} - \boldsymbol{bc} \neq \boldsymbol{0}$ is conformal.

(C) **ATTEMPT ANY ONE** **03**

(1) Prove that the composition of two bilinear maps is a bilinear map.

(2) Discuss the transformation $\boldsymbol{w = e^z}$ in polar form.

Que-4

(A) **ATTEMPT ANY ONE** **07**

(1) Define: Region & Radius of Convergence of Series.

Find region and radius of convergence for (i) $\sum_{n=1}^{\infty} n!\, z^n$ and (ii) $\sum_{n=1}^{\infty} \frac{z^n}{n!}$.

(2) Define: Analytic Function

State and prove Taylor's infinite series of an analytic function $\boldsymbol{f(z)}$.

(B) **ATTEMPT ANY ONE** **04**

(1) Show that $\boldsymbol{\frac{1}{z^2} = \frac{1}{4} + \frac{1}{4}\sum_{n=1}^{\infty}(-1)^n(n+1)\frac{(z-2)^n}{2^n}}$.

(2) Expand $\boldsymbol{\frac{e^z}{z(1+z^2)}}$ in Laurentz's series for $\boldsymbol{|z| < 1}$.

(C) **ATTEMPT ANY ONE** **03**

(1) Define: (i) Complex Sequence (ii) Power Series (iii) Absolutely Convergent Series.

(2) Expand $\frac{1}{1+z}$ in Maclaurine's series.

Que-5

(A) **ATTEMPT ANY ONE** **07**

(1) Define: Pole.

If $\boldsymbol{z_0}$ is the $\boldsymbol{m^{th}(m \geq 2)}$ order pole of a complex function $\boldsymbol{f(z)}$, then prove that $\boldsymbol{Res(f(z), z_0) = \frac{\varphi^{m-1}(z_0)}{(m-1)!}}$, where $\boldsymbol{\varphi(z) = (z - z_0)^m f(z)}$ is an analytic function and $\boldsymbol{\varphi(z_0) \neq 0}$.

(2) State the Cauchy's residue theorem and evaluate $\int_C \frac{e^z}{z(z-1)^2}dz$, $C: |z| = 2$.

(B) **ATTEMPT ANY ONE** **04**

(1) Evaluate: $\boldsymbol{\int_0^{\infty} \frac{1}{(x^2+1)^2} dx}$.

(2) Evaluate: $\boldsymbol{\int_0^{\pi} \frac{d\theta}{(2+cos\theta)^2}}$.

(C) **ATTEMPT ANY ONE** **03**

(1) Evaluate: $\int_0^{2\pi} \frac{d\theta}{\frac{5}{4}+sin\theta}$.

(2) Evaluate: $\int_0^{\infty} \frac{x^2}{(x^2+4)(x^2+9)} dx$.

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